Generalized Mach–Zehnder interferometer architectures for radio-frequency translation and multiplication: Suppression of unwanted harmonics by design

Ramón Maldonado-Basilio, Mehedi Hasan, Rabiaa Guemri, Frédéric Lucarz, Trevor J. Hall

ABSTRACT

A generalized array of N parallel phase modulators electrically driven with a progressive 2π/N phase shift is analyzed. For N-even, the equivalence of this configuration to parallel Mach–Zehnder architectures, and specifically the equivalence for N=4 to a dual parallel Mach–Zehnder modulator is shown. A simple approach to the design of this architecture that determines the static optical phase shifts required in each of the N parallel arms to suppress unwanted harmonics while maximizing the harmonics of interest is developed. The proposed design approach is validated by numerical simulations of N=4 and N=6 architectures with properly determined optical phase shifts. Optical single-side-band modulation (lower and upper) and frequency multiplication of an electrical drive signal with high suppression of unwanted harmonics is shown to be achievable.

Keywords: Photonic integrated circuits
Radio frequency photonics
Modulators

1. Introduction

Since the first experimental demonstration of integrated-optics circuits comprising four LiNbO₃-based phase modulator (PM) waveguides in the arms of splitters (input) and couplers (output) for 2 GHz optical frequency shifting [1], numerous configurations with differing optical and radio frequency (RF) parameters but equivalent circuit architecture have been extensively investigated [2–16]. To mention but a few, an improved version of [1] has been reported in [2] to implement optical frequency shift of a 10 GHz modulated carrier with –18.4 dB of carrier suppression. Radio-frequency attenuators and phase shifters are used to adjust the amplitude and phase of the input drives, and additional bias is required to adjust the optical phases. A configuration for 20 GHz single-side-band (SSB) modulation without optical carrier suppression that uses a 1 x 4 multi-mode interference (MMI) coupler to eliminate the requirement of static optical phase shifters has been proposed and validated with numerical simulations [3]. The structure of four parallel phase modulators has also been studied by simulation for optical frequency 8-tupling [4–6], 12-tupling [7], 16-tupling [8], and 24-tupling [5] schemes requiring no optical filters. An evolution of the configuration in [1] groups the four phase modulators into two pairs with electrical differential drive. Such an architecture is conceptually equivalent to a parallel pair of Mach–Zehnder modulators (MZM), each of them placed in the arms of an outer Mach–Zehnder Interferometer (MZI), and is known as a dual-parallel (DP)-MZM. The DP-MZM has become the standard circuit architecture to implement electro-optic frequency conversion, optical frequency multiplication, and SSB or in-phase/quadrature (IQ) modulation [9–16]. As an IQ modulator, it offers the direct implementation of quadrature phase-shift keying (QPSK), and the more general class of quadrature amplitude modulation (QAM) used in mainstream coherent optical communications links and radio-over-fiber (RoF) [13–16].

A selected list (only non-cascade and optical filter-less architectures are included) of publications describing essentially the same Generalized Mach–Zehnder Interferometer (GMZI) circuit adapted to a particular application is presented in Table 1. The great variety of differing optical and RF parameters that are set to tailor the GMZI architecture to different applications motivates the development of a systematic design approach to implement some of the above mentioned functionalities. Previously, electrical and optical parameters found by ad-hoc methods and to give the desired function have simply been stated. For the first time, this paper presents a simple systematic design procedure to determine the parameters required to deliver the desired function. The aim of the present work is therefore to analyze a generalized array of N-parallel phase modulators, each of them electrically driven with...
a progressive $2\pi/N$ phase shift, and to determine the configuration of optical weights (i.e., phase shifts) that suppresses specific unwanted harmonics while maximizing the harmonics of interest. For even $N$, pairs of differentially driven phase modulators may be grouped together and the architecture analyzed as $N/2$ parallel Mach–Zehnder modulators. In particular, for $N = 4$, the configuration is equivalent to the DP-MZM architecture. Moreover, although today LiNbO$_3$ is the most mature integration platform for DP-MZM, the proposed approach is most suited to Si- or III–V-based integration platforms given the development of linear electro-optic modulator technology. Recent demonstrations of traveling wave InAlGaAs-based electro-optic modulators, 1 cm long, 0.77 V drive voltage ($V_d$) and exceeding 67 GHz bandwidth [17]; athermal InP integrated twin IQ modulators with a total footprint of 1.2 mm $\times$ 11 mm for 56 Gb/s QPSK [18]; as well as strained rib-waveguide silicon MZMs [19]; and silicon-organic hybrid IQ modulators for 16-QAM at 112 Gb/s [20]; augur well that suitable modulator devices will be forthcoming. Furthermore, the coarse optical phase shifts can be realised with proper design by exploiting the intrinsic phase relations between the ports of the splitters and couplers used with the phase modulators. If required, fine tuning can be made by small footprint optical phase shifters occupying a design area of a few square micrometers [21]. Similarly, preset or variable trimming can be implemented to compensate for the imbalance in modulators [22] due to fabrication errors or drift, respectively.

2. Design approach

Consider the array of $N$ phase modulators in parallel shown in Fig. 1, where each modulator is driven electrically by a cosinusoidal waveform with a progressive phase shift in units of $2\pi/N$. Using the Jacobi–Anger expansion, the complex amplitude of the field at the output of each PM can be expressed as:

$$\exp\left[i \cos(\theta + p\frac{2\pi}{N})\right] = \sum_{q=-\infty}^{\infty} \exp\left[i pq \frac{2\pi}{N}\right] j_q(m) \exp(iq\theta)$$

where $m$ is the modulation index; $\theta = \omega t$ and $p2\pi/N$ are the dynamic and static phase of the cosinusoidal electrical drive signal; the positive integer $p$ denotes the index of the phase modulator in the array; $j_q(\cdot)$ is the Bessel function of the first kind with order $q$, and $i = \sqrt{-1}$ is the imaginary unit. The output of the combiner is

$$\sum_{p=0}^{N-1} a_p \exp\left(i p \frac{2\pi}{N}\right)$$

where the discrete Fourier transform

$$\tilde{a}_q = \frac{1}{N} \sum_{p=0}^{N-1} a_p \exp\left(i pq \frac{2\pi}{N}\right)$$

has entered the formulation. The sequence $a_p$ denotes the complex weight of each phase modulator, which is preferably uniform to minimize loss of energy, i.e., a phase shift. It is observed that the sequence $a_q$ is periodic with period $N$. The importance of Eqs. (2) and (3) can be understood by considering first the simplest case for the weights, which is $a_p = 1$ for $p = 0, 1, \ldots, N - 1$. The discrete Fourier transform term (3) may then be evaluated to yield:

$$\tilde{a}_q = \frac{1}{N} \left(1 - \exp(i2\pi q)\right) = \frac{1}{N} \exp\left[i q \left(\frac{N-1}{N}\right)\right] \frac{\sin(q\pi)}{\sin(q\pi/N)}$$

where the geometric series summation $\sum_{p=0}^{N-2} z^p = (1 - z^N)/(1 - z)$, with $z = \exp(i2\pi q/N)$, has been used. Applying L'Hôpital's rule to Eq. (4), it is found that $a_0 = 1$ for $q = 0$, and $a_q = 0$ for $q = 1, 2, \ldots, N - 1$. The frequency domain sequence therefore suppresses periodically all harmonics except those that are multiples of $N$. One may take advantage of the shift theorem by modifying a given set of weights...
by the application of a progressive phase factor with increment $-q_0 2\pi/N$. That is $a_p \rightarrow a_p \exp(-ipq_0 2\pi/N)$ and hence:

$$\tilde{a}_q = \frac{1}{N} \sum_{p=0}^{N-1} [a_p \exp(ip(q - q_0) 2\pi/N)] = \tilde{a}_{q-q_0}$$ (5)

The origin of the frequency domain sequence can be shifted therefore to position $q_0$ by a progressive phase shift of the light exciting the phase modulators with increment $-q_0 2\pi/N$. For the special case $a_p = 1$ and integer $q_0$, $a_{q_0} \rightarrow -a_{q_0}$ is zero for each order except $q = q_0 + rN$, where $r$ is an integer. The design approach hence simplifies to the determination of a configuration of coefficients that suppresses specific harmonic orders while it maximizes the harmonic orders of interest. An equivalent interpretation for the suppression function is found by expressing (3) as

$$a_q = (1/N) \sum_{p=0}^{N-1} [a_p z^p]$$, where $z = \exp(iq 2\pi/N)$ and then

$$a_q = f(\exp(iq 2\pi/N))$$ where

$$f(z) = \frac{1}{N} \sum_{p=0}^{N-1} a_p z^p = \frac{1}{N} \prod_{j=1}^{N} \left(1 - \frac{z}{z_j}\right)$$ (6)

The value of the suppression function $f$ on the unit circle determines the weighting of the harmonics generated by the phase modulators. Note that the zeroes $z_j$ of the suppression function may be placed anywhere in the complex plane. However, it is preferable that $|z_j| = 1$ in which case the distribution of zeros is constrained. A design approach based on the application of a suppression function to minimize unwanted harmonics wherever is needed constitutes one of the main assets of the GMZI architecture. Since the process does not depend upon the modulation index, the linearity of the $N$-parallel phase modulators array is maintained and each PM can be driven at moderate input powers. Furthermore, for an even number of phase modulators, there will be pairs of phase modulators in differential drive, leading to an equivalent parallel MZM interpretation.

3. Simulation results

The operation of the $N$-parallel phase modulator array designed according to the approach presented in this paper is verified by computer simulations using the Virtual Photonics Inc. (VPI) software package. For simplicity, arrays comprising four and six phase modulators are shown as illustrative examples, although the design method holds for any number of modulators. A continuous wave DFB laser diode set at a wavelength of 1550 nm with a line-width of 200 kHz and power of 10 mW is used as the optical input. Optical phase shifters are used to model the uni-modular complex coefficients $a_p$, whereas cascaded Y-branches are used as the input $1 \times N$ splitter and output $N \times 1$ coupler.

3.1. Array of four phase modulators

The schematic diagram of a generalized Mach–Zehnder interferometer consisting of an array of four phase modulators ($N = 4$) is shown in Fig. 2, where each pair of PMs has been arranged together as a MZM. The sinusoidal drive signals at the input of each phase modulator have a frequency of 10 GHz and progressive electrical phase shifts $\varphi_p$ equal to 0, $+\pi/2$, $+\pi$, and $+3\pi/2$, respectively. To achieve single-sideband modulation, the coefficient $q_0 = 0$ and the lowest order unsuppressed harmonics are $-9$, $-5$, $-1$, $+3$, and $+7$. In this case, the required optical weights $a_p$ also have a progressive phase shifts of $0$, $+\pi/2$, $+\pi$, and $+3\pi/2$, respectively. As illustrated in the insets of Fig. 2, the quadrature driving signals to each pair of MZM (inset A) generate a composite spectrum comprising an optical carrier with a main harmonic located 10 GHz off the carrier frequency plus secondary modulation harmonics (insets B and C). In line with the design approach developed above, constructive interference by the imposed optical phase shifts benefits the lower order harmonic located 10 GHz to the left of the optical carrier. Due to destructive interference, other harmonics including the optical carrier are suppressed (inset D). In the equivalent LiNbO$_3$-based DP-MZM configuration, the same SSB modulator can be achieved by biasing the inner MZMs and outer MZI at the minimum- and half-transmission points, respectively.

Following the same design approach, Table 2 summarizes the optical and electrical phase shifts required to achieve lower and upper single-sideband modulation ($q_0 = \pm 1$), frequency quadrupling ($q_0 = \pm 2$), and frequency octupling ($q_0 = 0$ with the drive level adjusted to suppress the carrier). In addition, the optical spectra obtained at the output of the four modulators array associated to the above mentioned functions are shown in Fig. 3(a)–(d), respectively. The frequency multiplication is obtained after the output of the proposed architecture is passed through a square-law photo-detector.

3.2. Array of six phase modulators

The schematic diagram of a generalized Mach–Zehnder interferometer consisting of an array of six phase modulators ($N = 6$) is shown in Fig. 4, where again each pair of PM has been arranged together as a MZM. The 10 GHz sinusoidal drive signals at the
input of each phase modulator have a progressive electrical phase shift of 0, π/3, 2π/3, π, 4π/3, and 5π/3, respectively. To achieve single-side-band modulation, \( q_0 = -1 \) and the lowest order unsuppressed harmonics are \(-13, -7, -1, +5 \) and \(+11 \). Similar to their electrical counterpart, the required optical weights \( a_p \) have a progressive phase shift of 0, \( π/3, 2π/3, \pi, 4π/3, \) and \( 5π/3 \), respectively. As illustrated in the insets B–D of Fig. 4, the \( N_{s}π/3 \) electrical phase shifts to each pair of PMs generates a spectrum that resembles a double side-band without carrier suppression profile plus higher order harmonics. The phase relations between the same order harmonics in each of the outputs B–D (see the time domain inset for B, C, and D) benefits the constructive interference of the lower order harmonic located at 10 GHz below the optical carrier frequency, whilst other harmonics including the carrier are suppressed. Since the structure is comprised of six PMs, the nearest unsuppressed higher order harmonic is obtained at \(-q = +5 \). In comparison to the four modulator array, the SSB output in the six modulator array provides better signal to unwanted harmonics ratio, which is +67 dB in comparison to 30 dB for the 4-modulator array at the same drive level. Table 3 summarizes the optical and electrical phase shifts required to achieve lower single side band (\( q_0 = -1 \)) and frequency sextupling (\( q_0 = \pm 3 \)). In addition, the optical spectra obtained at the output of the six modulator array associated to the above mentioned functions are shown in Fig. 5(a) and (b), respectively.

4. Discussion

As mentioned in the design approach addressed in section II and illustrated in the simulation results of section III, target applications of single-side-band or frequency multiplication are achieved by determining the configuration of coefficients that suppresses specific harmonic orders while maximizing the harmonic orders of interest. The performance of the proposed architecture hence relies on the accuracy of the electrical phase shifts and optical weights. By careful design and construction, the optical waveguides interconnecting the splitters, phase modulators, and combiners can be conveniently matched to minimize unwanted static phase shift imbalances, minimizing the need for trimming and ultimately static bias. Indeed, the drift of bias points in MZMs has been acknowledged as an unwanted issue and has led to a substantial volume of literature on the physical mechanisms responsible [23,24] and its alleviation by sophisticated stabilization mechanisms [25–28]. In case of applications requiring very high precision balancing to suppress undesired harmonics and intermodulation products in microwave photonic circuits, the trimming range can be made small facilitating semi-permanent and low power variable trimmers using, for example, ultra-compact thermo-optic variable attenuators and phase shifters. If the large static phase offsets are eliminated, the trimmer only needs to compensate for the residual imbalance and consequently both the range and the relative accuracy required of the trimmer is eased.

For a given target application, all necessary static optical phase shifts can also be defined by the intrinsic relative phase relations between the ports of the splitters and combiners, which can conveniently be implemented in an actual photonic circuit through multi-mode interference couplers (MMIs). In this regard, by using commercial software tools such as FimmWave from

Table 2

<table>
<thead>
<tr>
<th>Conditions and operation</th>
<th>PM-index</th>
<th>Phase shift</th>
<th>Optical shift</th>
<th>Electrical shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 = -1) ( {\ldots -9, -5, -1, +3, +7\ldots } ) SSB (lower)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( q_0 = +1) ( {\ldots -7, -3, +1, +5, +9\ldots } ) SSB (upper)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( q_0 = \pm 2) ( {\ldots -6, -2, +2, +6\ldots } ) Frequency quadrupling</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( q_0 = 0) ( {\ldots -8, -4, 0, +4, +8\ldots } ) Frequency 8-tupling</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 3. Optical spectra at the output of the four parallel phase modulators array for various design conditions.
PhotonDesign, it is found that in a 2×2 MMI (4.3 μm wide by 66.5 μm long, silicon-on-insulator wafer with 200 nm thick silicon over a 2 μm buried oxide layer), a phase deviation of 1° ± 1° off the ideal 90° phase difference between the two output ports occurs when the length of the MMI deviates by 1 μm ± 1 μm from its nominal design value. In addition, a deviation of 0.25% ± 0.25% off the ideal 50% optical power balance is obtained for the same scan in the length of the 2×2 MMI. As discussed in [5,29], the performance of single-sideband modulators and frequency multipliers making use of MMIs in conceptually similar architectures is highly tolerant to such variations originating from the design itself and ultimately from fabrication tolerances. For the extreme deviations of ±5° and ±5% discussed in [5], a signal to harmonic distortion ratio of over 30 dB is obtained, which is in agreement with the experimental demonstrations [9]. In applications requiring very high precision in the optical phases, miniature MZI operating as phase and amplitude adjusters can be considered to eliminate the residual phases and power imbalances required [30 and references therein]. Although today LiNbO3 is the most mature integration platform for DP-MZM, the proposed approach is most suited to Si- or III–V-based integration platforms given the development of linear electro-optic modulators technology. Recent demonstrations of traveling wave InAlGaAs-based electro-optic modulators, 1 cm long, low Vπ and exceeding 67 GHz bandwidth [17], athermal InP integrated twin IQ modulators with a total footprint of 1.2 mm × 1.3 mm for 56 Gb/s QPSK [18], as well as strained rib-waveguide silicon MZMs [19], and silicon-organic hybrid IQ modulators for 16-QAM at 112 Gb/s [20] augur well that suitable modulator devices will be forthcoming.

### Table 3

<table>
<thead>
<tr>
<th>Conditions and operation</th>
<th>PM-index</th>
<th>Phase shift</th>
<th>Optical</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) q0 = 1±{⋯−13, −7, −1, +5, +11, ⋯} Improved SSB (lower)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>π/3</td>
<td>π/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2π/3</td>
<td>2π/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>π</td>
<td>π</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4π/3</td>
<td>4π/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5π/3</td>
<td>5π/3</td>
<td></td>
</tr>
<tr>
<td>(b) q0 = ±3±{⋯−9, −3, +3, +9, ⋯} Frequency sextupling</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>π</td>
<td>π</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2π/3</td>
<td>2π/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>π</td>
<td>π</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4π/3</td>
<td>4π/3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5π/3</td>
<td>5π/3</td>
<td></td>
</tr>
</tbody>
</table>

### 5. Conclusion

A systematic design method for the suppression of unwanted harmonics produced by parallel phase modulator array circuits for frequency translation and multiplication has been presented. The analyzed configuration comprises N-parallel phase modulators electrically driven by cosinusoidal waveforms with a progressive 2π/N phase shift. The method is particularly suited to photonic integrated circuit implementations. For N = 4, the analyzed circuit is conceptually equivalent to the DP-MZM architecture available in LiNbO3 technology. On the other hand, improved implementations of some functions can be achieved for a larger number of phase modulators, such as the IQ/single side band modulation in the analyzed example of six phase modulators. The method should also prove valuable in improving the performance of parallel phase modulator circuits implemented on Si and III–V integration platforms.

**Fig. 4.** Schematic diagram of a generalized Mach–Zehnder interferometer consisting of an array of 6 phase modulators in parallel arranged in pairs of MZMs. Insets A–E show the signals obtained at the output of the PM1, upper, middle, and lower MZM, composite time domain signal, and overall output. The x-axis in B–E is shifted to the center frequency of the optical carrier. In A, the phase shifted RF drive for PM2 is shown in dotted line.

**Fig. 5.** Optical spectra at the output of the six parallel phase modulators array for various design conditions.
Acknowledgments

Authors acknowledge Natural Sciences and Engineering Research Council of Canada (NSERC) for their support through a Strategic Project Grant. Trevor J. Hall is grateful to the Canada Research Chair (CRC) Program for their support of his CRC-I in Photonic Network Technology.

References